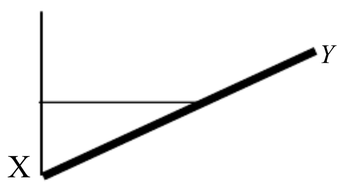
MECHANICS (C) UNIT 2 TEST PAPER 6

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

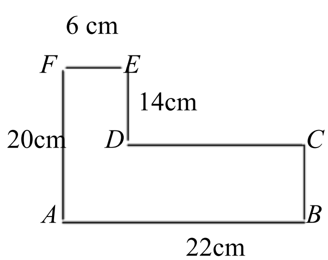
1. A ship, of mass 5000 tonnes, is moving through the sea at a constant speed of 15 km h^{-1}.
   (i) Calculate the momentum of the ship, in the form $a \times 10^n$, where $0 < a < 10$ and n is an integer. State the units of your answer. [2]
   Given that there is a constant force of magnitude 4000 N acting against the ship's motion due to air and water resistances,
   (ii) find the rate, in kW, at which the ship's engines are working. [3]

2. Two small smooth spheres P and Q are moving along a straight line in opposite directions with the same speed u, and collide directly. Immediately after the impact, the direction of P's motion has been reversed and its speed has been halved.
   The coefficient of restitution between P and Q is e.
   (i) Express the speed of Q after the impact in the form $au(be + c)$, where a, b and c are constants to be found. [4]
   (ii) Deduce the range of values of e for which the direction of motion of Q remains unaltered. [3]

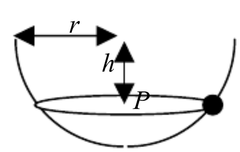
3. A small block of wood, of mass 0·5 kg, slides down a line of greatest slope of a smooth plane inclined at an angle α to the horizontal, where $\sin \alpha = {}^2/_5$. The block is given an initial impulse of magnitude 2 Ns, and reaches the bottom of the plane with kinetic energy 19 J.
   (i) Find, in J, the change in the potential energy of the block as it moves down the plane. [2]
   (ii) Hence find the distance travelled by the block down the plane. [3]
   (iii) State two modelling assumptions that you have made. [2]

4. 

   A uniform rod XY, of length $2a$ and mass m, is connected to a vertical wall by a smooth hinge at the end X. A horizontal light inelastic string connects the mid-point of XY to the wall and the rod is in equilibrium in this position.
   (i) Draw a diagram to show all the forces acting on the rod. [3]
   Given that the tension in the horizontal string is of magnitude $2mg$,
   (ii) find the angle which XY makes with the vertical. [5]

5. 

   The diagram shows a uniform lamina $ABCDEF$.
   (i) Calculate the distance of the centre of mass of the lamina from (a) AF, (b) AB. [6]
   The lamina is hung over a smooth peg at D and rests in equilibrium in a vertical plane.
   (ii) Find the angle between CD and the vertical. [3]

6. The diagram shows a particle P of mass m kg moving on the inner surface of a smooth fixed hemispherical bowl of radius r m which is fixed with its axis vertical. P moves at a constant speed in a horizontal circle, at a depth h m below the top of the bowl.


   (i) Show that the force exerted on P by the bowl has magnitude $\dfrac{mgr}{h}$ N. [4]
   (ii) Find, in terms of g, h and r, the constant speed of P. [5]

7. A projectile is fired with speed 8 ms^{-1} from a point O on a horizontal plane, at an angle elevation α. It moves under gravity such that, at time t seconds after projection, it is at the point with coordinates $(x,\ 4t - 4{\cdot}9t^2)$ relative to O, where the x and y directions are respectively horizontal and vertically upwards.

(i) Show that $\sin\alpha = 0{\cdot}5$ [3]

(ii) Hence find x in terms of t. [2]

(iii) Show that $y = \dfrac{x}{\sqrt{3}} - \dfrac{gx^2}{96}$ [3]

(iv) Deduce the value of x when the projectile returns to the horizontal plane. [2]

On another occasion, the equation of the projectile's path is $y = \dfrac{3x}{4} - \dfrac{gx^2}{32}$.

(v) Find the angle of projection and the initial speed of the projectile. [5]

MECHANICS 2 (C) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1. (i) $5\,000\,000 \times (15\,000 \div 3600) = 2{\cdot}08 \times 10^7$ Ns or kg m s^{-1} — M1 A1

(ii) $P = 4000 \times (150 \div 36) = 16{\cdot}7$ kW — M1 A1 A1 — 5

2. (i) $[v - (-u/2)] / (-u - u) = -e$ — M1 A1

$v + \frac{1}{2}u = 2ue$ Speed $= \pm\ \frac{1}{2}u(4e - 1)$ — M1 A1

(ii) If $v < 0$ then $4e - 1 < 0$, so $0 < e < \frac{1}{4}$ — M1 A1 A1 — 7

3. (i) P.E. lost = K.E. gained $= 19 - \frac{1}{4} \times 0{\cdot}5 \times 4^2 = 15$ J — M1 A1

(ii) $0{\cdot}5gh = 15$ $h = {}^{30}/_{g}$ $d = h \div \sin\alpha = {}^{75}/_{g}$ $= 7{\cdot}65$ m — M1 A1 A1

(iii) Modelled block as particle; ignored air resistance — B1 B1 7

4. (i) Diagram showing weight, tension, two components of reaction or single reaction force at X — B3

(ii) $M(X)$: $2mg\,a\cos\theta = mg\,a\sin\theta$ $\tan\theta = 2$ $\theta = 63{\cdot}4^0$ — M1 A1 A1 M1 A1 — 8

5. (i) (a) $132(11) + 84(3) = 216\,\bar{x}$ $\bar{x} = 7{\cdot}89$ — M1 A1 A1

(b) $132(3) + 84(13) = 216\,\bar{y}$ $\bar{y} = 6{\cdot}89$ — M1 A1 A1

(ii) $\tan\alpha = 0{\cdot}89 \div 1{\cdot}89 = 0{\cdot}471$ $\alpha = 25{\cdot}2^0$ — M1 A1 A1 — 9

6. (i) Reaction R acts on P towards centre of sphere, at θ to vertical — M1

where $\cos\theta = h/r$ $R\cos\theta = mg$, so $R = \dfrac{mgr}{h}$ — B1 M1 A1

(ii) Resolve towards centre : $R\sin\theta = mv^2/(r\sin\theta)$ — B1

$v^2 = (mgr^2 / h)(\sin^2\theta / m) = (gr^2 / h)(r^2 - h^2) / r^2$ — M1 A1 A1

$v = \sqrt{[g(r^2 - h^2) / h]}$ — A1 — 9

7. (i) $y = (8\sin\alpha)t - 4{\cdot}9t^2 = 4t - 4{\cdot}9t^2$ (given), so $\sin\alpha = \frac{1}{2}$ — M1 A1 A1

(ii) $x = 8t\cos\alpha = (4\sqrt{3})t$ — M1 A1

(iii) $y = \dfrac{4x}{4\sqrt{3}} - \dfrac{gx^2}{2(4\sqrt{3})^2} = \dfrac{x}{\sqrt{3}} - \dfrac{gx^2}{96}$ M1 A1 A1

(iv) When $y = 0$, $\dfrac{x}{\sqrt{3}} = \dfrac{gx^2}{96}$ $gx = {}^{96}/_{\sqrt{3}}$ $x = 5{\cdot}66$ M1 A1

(v) $\tan\alpha = 0{\cdot}75$ $\alpha = 36{\cdot}9^0$ M1 A1

$2u^2\cos^2\alpha = 32$ $u^2({}^{16}/_{25}) = 16$ $u = 5 \text{ ms}^{-1}$ M1 A1 A1 15